Note: The following lesson plan assumes the students have mastered the following prerequisites:

- Modeling linear algebraic expressions using algebra tiles including:
  - Modeling the distributive property
  - Use of the additive inverse to create zero pairs
  - Simplifying expressions using algebraic addition of like terms
- Modeling integer algebraic addition with colored markers

1 Teaching objectives

This lesson will extend the students current understanding of modeling integer algebraic addition and algebraic expressions using algebra tiles to solving linear equations with integer solutions. The approach will feature an emphasis on keeping the equation in balance through the use of the additive inverse and the subsequent use of the zero pair concept. This addresses 2a (Solve and check multistep linear equations) of the Mississippi 2007 revised curriculum.

2 Instructional Activities

a) Divide the class into groups of 4 students each. Any grouping that the teacher has had success with is acceptable, however an academically diverse grouping is recommended.
b) Ask each group to select a group leader, a resource person, a recorder person, and a presenter.
c) Have the resource person from each group come to the front and get the following resources for each member of their group:
   - 1 set of algebra tiles
   - 1 twelve inch ruler
   - 2 sheets of copying paper
   - 1 sheet of lined paper

In addition have them pick up one additional set of algebra tiles.
d) Have the resource person distribute the materials to each member of their group and place the extra set of algebra tiles to the side of their table. These are to be used if a particular equation requires more tiles than are in a standard set of tiles.

e) On the overhead projector write two algebraic expressions for the students. Put one expression on the left side of the screen and the second expression on the same line (horizontally) but on the right side of the screen and ask the students to build the two models and place them on their white copying paper just as they appear on the screen. Note these two expressions represent the left side and the right side of the example equation, BUT do not introduce this to the students at this point.

f) Ask the presenter from two different groups to come to the front and put their model of the two expressions on the overhead immediately below the written expressions.

g) Congratulate the class for remembering how to correctly model algebra expressions.

h) Write an equal sign on the overhead between the two expressions. Initiate a discussion with the class as to what is meant by the equal sign. Make sure the discussion brings out the following points:
   - An equal sign is a powerful mathematical symbol that means that whatever type of expression (algebraic or arithmetic) appears to the left of the sign is exactly equivalent (the same mathematically) to the expression to the right of the sign.
   - Due to this equivalency there is a balance established such that if anything is added to either side of an equal sign, the exact same thing must be added to the opposite side.
   - The equal sign is the symbol that makes the two expressions into an equation.
   - The equal sign divides the equation into a LS (left side) and into a RS (right side).

i) Place your ruler on the overhead vertically between the models of the two expressions. Tell the class that we will use our rulers to represent our equal signs to divide our equation into a LS and a RS. Instruct the class to do likewise.

j) Ask the class simplify each side of the equation. Have the presenter from two different groups come to the overhead and show how they simplified one side of the equation. Make sure that the students use the terms:
   - additive inverse
   - zero pairs
   - like terms

k) Start a heading on the white board (or blackboard):
   “Steps in Solving a Linear Equation”

l) Tell the students to put this in their notebooks and that step 1 is to simplify each side of the equation and write this on the white board.
m) Have the class determine the number of x’s on the LS and the number of x’s on the RS remind the class that the answer will be a signed integer. Have the presenter from each of two different groups come to the front and show how they determined the signed number of x’s on each side of the equation.

n) Explain to the class that the next step in solving equations is having x’s on only one side of the equation, and not just any x’s, but only positive x’s. Lead a discussion as to how to accomplish this making sure that the discussion makes the following points:

- If only one side of the equation has x’s and they are positive, then we can skip this step.
- If only one side of the equation has x’s and they are negative then we add the additive inverse number of positive x’s to both sides (to maintain equation balance) and then simplify the side that has the zero pairs of x’s.
- If there are positive x’s on both side of the equation, then we start with the side that has the least number of x’s and add to that side the additive inverse of the x’s AND in order to maintain balance we put the same number of negative x’s on the other side of the equation.
- If there are positive x’s on one side and negative x’s on the other side, we add the additive inverse to the x’s on the negative side making sure we maintain the equation balance by also adding the same number of positive x’s to the side containing the positive x’s.

Summarize the above by stating that the second step in solving an equation is to eliminate the x’s on the side of the equation that has the least number of x’s (taking sign into account) by adding the additive inverse to both sides of the equation.

o) Write on the white board “Step 2 – Use the additive inverse property to eliminate the variable (x in this case) from the side of the equation that has the least number of variables.”

p) Ask the class to perform step 2 in their groups and when they have completed it, ask presenters from two different groups come to the overhead and show how they performed step 2. Make sure the students see that there are only green positive x’s and that they are only on one side of the equation.

q) Now is a good time to tell the students to take a deep breath; that they have done some VERY GOOD work and are almost home.

r) The third step to solving an equation is to once again use the additive inverse, but this time it will be used to remove any unit tiles that are on the same side of the equation as the x’s. Make sure you use the word “balance” when you ask them to add the additive inverse to both sides of the equation. Also remind them that they must simplify both sides of the equation at the end of each step. Have the presenter from two different groups come to the front and demonstrate how they used the additive inverse on both sides and then how they subsequently simplified both sides. Make sure you mention that if there are no unit tiles on the same side of the equation that contains the x’s that this step is unnecessary and can be skipped. There should now be green x tiles on
one side of the ruler and either all red or all yellow unit tiles on the other side of the ruler. It doesn’t matter which side is which as long they are on opposite sides.

s) Write on the white board “Step 3 – use the additive inverse (if necessary) to remove any unit tiles that are on the same side of the equation as the x’s “.

t) Say “Hurray, we’re at the last step “or some phrase that conveys that the task is almost completed.

u) Tell the students to turn their x tiles into the vertical position, then one at a time distribute the unit tiles equally to the x tiles until all the unit tiles have been used up and each x has the same number of unit tiles beside it. The number of tiles that each x tile receives is the solution to the equation. Ask a presenter from one group to come to the front and demonstrate how to distribute the unit tiles and what value for the solution of x was found.

v) Write on the white board “Step 4 – distribute the unit tiles evenly one at a time to the x tiles until they are all used up. The solution is the number of unit tiles that EACH x tile received.

w) Have the recorder draw the 4 steps of the equation solution on the lined sheet of paper and mark it as Example 1.

x) Have the students practice this with several (6 is a good number) simple (integer solutions) linear equations and have the recorder copy their group’s solution on the lined paper, put all the group’s names and the date on it and hand it in at the end of the activity.

3 Materials and Resources

a. 5 sets of algebra tiles for each group
b. 1 ruler 12 inch for each group
c. 1 sheet of lined paper for each group
d. 2 sheets of white copy paper for each group
e. This lesson is all original content

4 Assessment

This lesson will be assessed in two distinct stages.

- The preliminary assessment will be the day of the lesson. Each group will be assigned 30 possible points. 28 points for correctly handing in Example 1 and the six practice equations (4 points each based on 1 point for each correct step) and 2 points for neatness, names appearing correctly, and the proper date.
- The second assessment will be a test that will take place at least 2 days later (this allows one day for remedial work if the preliminary assessment indicates it is required or if a student feels that he/she needs additional help) and will consist of 10 equations to be solved, the answers will require detail for all 4 steps and will
be scored 4 points per equation, for a total of 40 points plus 2 additional points for correct date, title, name on the answer sheet.
Appendix A

Suggested Equations

Class Work

a) \( X + 2 = 3X - 6 \)
b) \( 3( X - 2) = X + 4 \)
c) \( 2( X + 1) = 3(1 - X) + 9 \)
d) \( 3 - (X - 4) = 3X - 1 \)
e) \( 2X + 3 + X = 5 + X \)
f) \( 2(X + 1) + 3(X - 2) = 3X + 2 \)
g) \( 4(X + 1) - 3 = 2X - 3 \)

Assessment

a) \( X - 2 = 3X + 6 \)
b) \( 3 - 2X + X = 3X + 7 \)
c) \( 3(X + 2) = X - 4 \)
d) \( 3(1 + X) - 9 = 2(X - 1) \)
e) \( 4X = -2X + 6 \)
f) \( 3X - 2 = 3(X + 2) + 2(X - 1) \)
g) \( -2X + 3 = -5X \)
h) \( 2X - 3 + X = X - 1 \)
i) \( 2X - 3 = 4(X - 1) + 3 \)
j) \( 3X + 4 = 7X \)
Appendix B

Solution Key

Class Work

a) 4
b) 5
c) 2
d) 2
e) 1
f) 3
g) -2

Assessment

a) -4
b) -1
c) -5
d) 4
e) 1
f) -3
g) -1
h) 1
i) -1
j) 1