Algebra/Geometry Institute Summer 2008

Faculty Name: Anthony deBlois

School: Grenada High School

Grade Level: 9th -12th grade

Title: A Triangle Expansion

1.   Teaching
    Objectives: The student will identify the coefficient of a term in an algebraic expression.
    The student will use the foil method to multiply two binomials together.
    The student will use Pascal's triangle to raise (x + 1) to the *n* power.

2. Instructional
    Activities: I. (The desks will be divided into groups so that the class is divided into an
        equal number for each group.) The teacher will begin the lesson by
        writing the word, FOIL, vertically on the board. The teacher will write
        the words, *first, outside, inside, last,* to the left of each letter in the word
        FOIL, respectively. Next, the teacher will ask 4 volunteers to come to the
        front of the room and line up in a straight line facing the class. (See
        Diagram A.) [5 min]

        A. The teacher will review the acronym, FOIL which was taught yesterday
            by asking a student who is sitting down, "Of the students standing in
            line, which student is furthest to your left?" After the correct response is
            given, the teacher will tell the class, "This person is first in line," and
            write the person's name on the board to the right of the letter F.
        B. The teacher will ask another student who is sitting down, "In the student
            line, which student is last in line?" After the correct response is given,
            the teacher will tell the class, "This person is last in line," and write the
            person's name on the board to the right of the letter L.
        C. From the class' point of view, the teacher will then ask the two students
            furthest to the left of the line to each simultaneously walk three steps
            sideways to their right. Next, the teacher will ask the two students
            furthest to the right of the line to each simultaneously walk three steps
            to their left. This line adjustment will divide the student line into two
            lines. (See Diagram A.)
        D. The teacher will say to the class, "Now, because we have two lines each
            made of two people, we have two first persons and two last persons.
            The first person of the line furthest to the left has already been labeled
            as the first person, the second person of the line furthest to the right has
            already been labeled as the last person. But, there are two more names
            to go to the right of F and L."

E. The teacher will ask the class, "Whose names do we need to add to the names already listed on the board by the letters F and L?"
   1. In the two-person line furthest to the right, the first person would be the person to the left.
   2. In the two-person line furthest to the left, the last person would be the person to the right.
F. The teacher will call on a student to write the two names that go to their respectful place by the letters F and L. (If needed, the teacher will assist the student who is called on.)
G. Next, the teacher will ask a student standing in line, "Which two of you are the two outside people of the four of you in line?" After the correct response is given, the teacher will write on the board the two names beside the letter O.
H. Lastly, the teacher will ask a student standing in line, "Which two of you are inside the line of the two outside people?" After the correct response is given, the teacher will write on the board the remaining two names beside the letter I.
I. In conclusion, the teacher will tell the class to give those who participated a hand. The teacher will tell the remaining students who are standing that they can sit down.

II. The teacher will begin the next activity by asking a volunteer student to hand out Worksheets I and II. [15 min]

   A. After a handout is given to each student, the teacher will begin by asking the class, "By using the acronym, FOIL, that we created as a class on the board, we will raise the binomial $(x + 1)$ to the second power by multiplying $(x + 1)$ times $(x + 1)$." The teacher will demonstrate problem 1 from Worksheet I on the overhead. The teacher will ask the students to use the FOIL method for problems 2 and 3 by working in their groups. (See Worksheet I.)
   B. As the students work in their groups, the teacher will remind the students of the definition of a binomial. A binomial is "a polynomial with two terms or a sum or difference of two monomials."
   C. The teacher will ask the class if they notice any patterns in the coefficients in problem 5. (See problem 5 on Worksheet I.)

III. The teacher will introduce the next activity by asking the class to raise their hand and recall the answer to the question, "What is a coefficient?" (The teacher will remind the students that this concept has already been taught.) The teacher will have a student who raised his/her hand answer. [10 min]

   A. After the student answers, the teacher will reiterate that the coefficient is "a number in front of a variable" by writing the definition on the overhead.
   B. After the definition of coefficient is re-stated, the teacher will write out

the trinomial, "$3x^2 + x + 5$" on the overhead. The students will be asked to identify the coefficients for that particular trinomial by the teacher asking, "what is the coefficient for x^2 ?", and "what is the coefficient for x?"

    C. The teacher will review coefficients with ten more examples on flashcards. (See Attachment A.) [10 min]

IV. The teacher will begin the last activity by giving some history about Pascal's Triangle and then have students work on Worksheet II.

    A. The teacher will give some history on Pascal's Triangle by asking the class, "How long ago do you all think Pascal's Triangle was first used?" After the students answer, I will tell them the answer is as early as the 1100's. Next, I will say, "even though Pascal's triangle was used before the time of Blaise Pascal, it is still named after Blaise Pascal because he was known for intensely using it to find several patterns. Class, the patterns we are looking for today are the coefficients "x" to each binomial expansion."

    B. Secondly, the teacher will ask the students to read the directions to Worksheet II and answer the questions. (See Worksheet II.) [20 min]

    C. The teacher will help the student groups get started with Worksheet II by going over Transparency Aid for Worksheet II. (See Transparency Aid for Worksheet II.)

    D. The teacher will lead a discussion about the coefficients in the binomial expansion and Pascal's Triangle by discussing the Transparency Aid for Worksheet II.

    E. The teacher will take up Worksheets I and II for a grade.

3A. Materials: Pencil
                  Attachment A
                  Worksheets I and II
                  Transparency Aid for Worksheet II
                  Diagram A
                  Overhead
                  Flash Cards
                  Notebook

3B. Resources: "What is a coefficient?" http://mathwizz.com/algebra/help/help4.htm. Last viewed on June 12, 2008.
               "What is a binomial?"   http://en.wikipedia.org/wiki/Binomial. Last viewed on June 12, 2008.
               Green, Thomas M. and Hamberg, Charles L. *Pascal's triange*. Palo Alto, CA, Dale Seymour Publications, 1986, pp. 1-2, 10-11.

4. Assessment: The teacher will observe and correct, if needed, students' responses in Activity I and students' work on Worksheets I and II. The teacher will have students

pass their completed work on Worksheeet I to another student and let the students grade each other's work. The teacher will have the students turn in Worksheet I and II for the teacher to grade.

1. Find $(x + 1)^2$ using the foil method and simplify.

2. Now, find $(x + 1)^3$ and simplify: (Note: just multiply the binomial expression "x + 1" to the answer given in number one above.)

3. Lastly, find $(x + 1)^4$ and simplify: (Note: multiply the binomial expression "x + 1" to the answer already given in number 2 since three of the expressions have already been multiplied together.)

   $(x + 1)(x + 1)(x + 1)(?)$

4. List the coefficients and constant(s) in order to answer 1-3.

   1. _____   3. _____
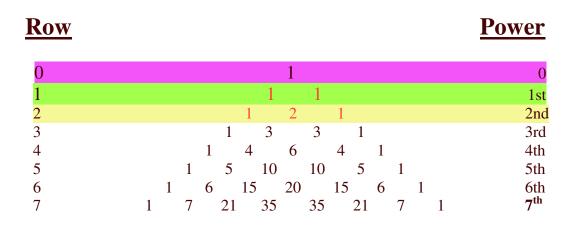
   2. _____

5. What patterns do you notice from number 4?

(Beginning with group 1, ask each student from each group in order one question from problems 1-10 until each student has answered once, then start over with group 1 until all questions have been answered.)

1. $5x + 4y$
   a. "What is the coefficient of x?"
   b. "What is the coefficient of y?"

2. $-21x^3 - 3x + 1$
   a. "What is the coefficient of x^3?"
   b. "What is the coefficient of x?"

3. $7a - 2b$
   a. "What is the coefficient of a?"
   b. "What is the coefficient of b?"

4. $8 + x$
   a. "What is the coefficient of a?"
   b. "What is the coefficient of b?"

5. $2a + (-3b) + 5$
   a. "What is the coefficient of a?"
   b. "What is the coefficient of b?"

6. $7m - 2n + (-2p) - (-q) + 1$
   a. "What is the coefficient of m?"
   b. "What is the coefficient of n?"
   c. "What is the coefficient of p?"
   d. "What is the coefficient of q?"

7. $6x^4 + x^3 - 2$
   a. "What is the coefficient of x^4?"
   b. "What is the coefficient of x^3?"

Looking at a part of Pascal's triangle below, the second row has the coefficients 1, 2, and 1 for a trinomial, $x^2 + 2x + 1$, respectively.

Row   Power

Row												Power
0						1						0
1					1		1					1st
2				1		2		1				2nd
3			1		3		3		1			3rd
4		1		4		6		4		1		4th
5	1		5		10		10		5		1	5th
6	1	6		15		20		15		6	1	6th
7	1	7	21		35		35		21	7	1	**7th**

*Example 1: The numbers on row 1 are the coefficients for the binomial below.

Row 0 = $x^0 = 1$ (with the 1 shown above)

**Row zero represents the constant 1, because it only has one number, 1.

*Example 2: The numbers on row 1 are the coefficients for x and the constant 1 for the binomial $(x + 1)^1$.
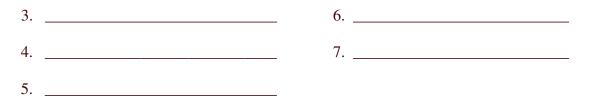
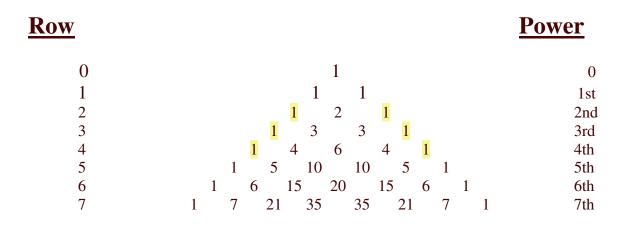Row 1 =   x + 1   (with 1   1 shown above)
            $(1x^1 + 1x^0)$

*Example 3: The numbers on row 2 are the coefficients for x^2, x, and the constant 1 for the trinomial $(x + 1)^2$:

Row 2 = $x^2 + 2x + 1$ (with 1   2   1 shown above)
            $(1x^2 + 2x^1 + x^0)$

Give the binomial expansion for row 3 thru row 7.

3. _____   6. _____

4. _____   7. _____

5. _____

TRANSPARENCY AID FOR WORKSHEET II

Row   ## Power

Row															Power
0							1								0
1						1		1							1st
2					1		2		1						2nd
3				1		3		3		1					3rd
4			1		4		6		4		1				4th
5		1		5		10		10		5		1			5th
6	1		6		15		20		15		6		1		6th
7	1		7		21		35		35		21		7	1	7th

Notice that the patterns of the coeffcients from Worksheet I begin with 1 and end with 1 and are also **palindromes.

Row 0 — $1x^0 = 1$

Row 1— $1x^1 + 1x^0$

Row 2— $1x^2 + 2x^1 + 1x^0$

**THE HIGHEST EXPONENT IS 2, SO THE COEFFICIENTS FOR THIS TRINOMIAL CAN BE FOUND ON ROW 2.

Row 3—

Row 4—

Row 5—

Row 6—

Row 7—